

- Heading: FATIO DE DUILLIER, Nicolas (1664–1753)
- Title: **Navigation improv'd** being chiefly the method for finding the latitude, at sea as well as by land, by taking any proper altitudes, together with the time between the observations: the whole made easy by a new trigonometrical rule. And the means how to refresh the air in the hold of a ship or of a man of war: and how to measure the number of waves, that pass under a ship during any given time; in order to make a truer estimation of the ship's way: and to have likewise thereby a sort of time-keeper.
- Imprint: **London:** printed in the months of April and May, 1728
- Collation: Folio: [pi]² ([pi]1 + A–B²), 6 leaves, pp. [4] 5–12. Author's manuscript note dated 5 June 1728 on verso of titlepage (see below).
- Plates: 1 folding engraved plate.
- Condition: 359 x 232mm. Some light dustoiling; plate waterstained.
- Binding: Disbound; evidence of original stab-stitching.
- Provenance: Probably William Jones (1675–1749) with annotations which appear to be in his hand; Earls of Macclesfield.
- References: ESTC T58837.
- Price: **£3,500**
- Edition: First (only) edition.

Fatio's method of finding latitudes, and other improvements to navigation, written up in 1716 after successful trials at sea but not previously published. Evidently a private publication (with no publisher's imprint) this was probably only presented to a small number of people and the British Library copy, like the present copy, has a note in Fatio's hand on the verso of the titlepage offering to demonstrate his method at sea. In the present copy this reads: 'London June the 5th 1728. I shall be further willing, upon proper Encouragement, to go to Sea a short Voyage to demonstrate and teach the Method which I do here propose; As, for instance, a voyage to Gottenburg, Copenhagen, or Portugal. N. Fatio.'

Fatio was a close friend to Newton and was involved in the Newton–Leibniz dispute over priority in the invention of the calculus in which William Jones was also a key player. This copy must have been given to Jones by Fatio. Jones' was tutor to the Earl of Macclesfield to whom he left his mathematical books.

In the preface, Fatio mentions his invention of jewelled bearings for watches for which he was awarded a patent in 1704, which he was forced to abandon because of opposition from the clockmakers company. The method of refreshing the air between decks is also briefly mentioned in the preface but not in the text.

Apart from the method of determining latitude from extra-meridian

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altitude and elapsed time, the text deals with a method of measuring the ship's way from the number of wave crests passing under the ship in a given time and their wavelength.

ESTC records copies at the British Library, Bodleian Library, Royal Society and New York Public Library. Not on ECCO.

Literature: Taylor, *Mathematical practitioners of Hanoverian England* pp. 117–8; Scott Mandelbrote in *ODNB*.

NAVIGATION IMPROV'D:

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METHOD

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A T

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A N D

To have likewise thereby a sort of Time-keeper.

By NICOLAS FACIO DUILLIER, R. S. S.

L O N D O N:

Printed in the Months of April and May, 1728.

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London June the 5th 1728.

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N. Pacio

P R E F A C E.



AMONG the many useful things, which Providence and my long and too unfortunate and ruinous Study of the Mathematicks has enabled me to find for the Improvement and Safety of Navigation, I shall chuse to publish now chiefly one, and to point out at some few more: By which I shall invite the Generosity and Justice of this flourishing Nation, in pursuance of the Acts of Parliament that are in force, to demonstrate, if they think fit, how far they were in earnest when those Promises and Invitations were made, which make me now to apply in this manner to the Public.

Had not I buried, in consequence of these Inquiries, by far the best part of my Substance, especially in introducing and establishing of Jewel-Watches, through all manner of Difficulties and Oppositions; for if I knew that Common Watches might do, I knew also that Jewel-Watches would do better; Surely I had, with the greatest Pleasure imaginable, published early and freely what I knew would be of general Advantage. But as I thought it would be but just, that, if the Public reaped the great Benefit arising from what I had found in this kind, I should, at least, be no loser by it; I confess I have only, from time to time, made, during these last five and twenty Years, some Attempts to try whether, after so great a Charge in sowing too freely, I could be sure of a Harvest, or of recovering the Losses and Damages which I had incurred, or at least a part of them, if I communicated or published the whole. But I found nothing that promised any Certainty of my being countenanced or encouraged any ways; but sometimes, and too often, quite the contrary. A melancholy Scene for one who reckons himself now the worse at least by three thousand Pounds, for having applied himself to those very Inquiries, and that too with uncommon Success!

However, seeing no Hopes of compassing the public Ends which I proposed to myself, unless it be done in my lifetime: This requires I should now make haste, and I set about it as follows.

I shall just hint, in the first place, what I found very lately, to wit, That there is an easy Method for refreshing continually the Air in the Hold and between Decks, in a Ship at Sea, by one or more large Bellows or Air Pumps playing continually by the force of the Ships rowing, or dipping, or ballancing itself any way. By the like Method, one or more Pumps, nay even single Canals or Tubes, may work of themselves, for pumping the Water out of a Ship. How many Lives might thus have been spared, even during these few last Years, let them judge that understand Physick, or have been, for instance, in the Royal Fleet, between the Tropicks for some Years. And thus much is enough for discovering the thing in general. But the Execution of it I leave to every ones own Choice and Convenience.

Secondly, I shall just mention a Method for finding at Sea, by Observation, the Measure of the various Swiftnes and Bigness of Waves. The same may be found with some degree of Exactness, by a just Theory also. And, if this be done, or even before it be done, some considerable Advantages will arise from an Instrument shewing continually upon a Dial Plate, by different Needles, or by Holes in the Plate, how many Thousands, and Hundreds, and Tens, and Units of Waves have passed under the Ship during any given Interval of Time, suppose an Hour or a Day &c, the Obliquity of the Course of the Ship to the Waves being observed also. Now that Interval of Time will be known, by any good Common or Jewel-Watch.

The Theory of Waves, as to their Bigness and Swiftnes, is exceeding difficult to be found. It has escaped the Industry and Sagacity of Sir Isaac Newton himself, tho' he has handled expressly this Matter: For he makes the Waves much smaller than I have found them, both by actual Observations at Sea, and by a Theory grounded upon some truer Principles, which that great Man did overlook. But this may be treated of more fully in another Place, together with some other things relating to the Perfection of Navigation: And then will be a properer time to write those minute Directions, that may serve best for mean Capacities. But now my chief Business is to convince Them that are more learned, and Them also who, if they but please, can want neither Power nor Methods, to bring these Inventions into use.

Waves, if they be once settled, are exceeding regular; and, while they continue of the same Bigness, they may even serve for a good Time-keeper. Upon which account, if in a Ship or Vessel there be no Watch or Clock, yet sometimes the Meridian Altitude of the Sun may be found on such Days as the Sun is seen but now and then between the Clouds, though there be no possibility of coming then by a Meridian Observation.

Let Q, R, S, Fig. VII. Represent in a large Abacus, or graduated Frame or Instrument, three or more Places in the heavenly Sphere, where the Sun's Altitude was observed: But let not those Places be too far from Noon. Therefore, upon the Horizontal Line Q T V, take or conceive the Segments Q T, T V proportional to the Number of Waves that have passed under the Ship between the Observations. Make T R and V S perpendicular to Q T V. Through the Points Q, R, S, &c. draw the uniform Curve or the Arc Q R S. The remotest Point in the Curve Q R S, from the Basis of the Abacus, shall give the Sun's Meridian Altitude for that Day. This Abacus may be engraved on purpose on a Board or a Slate, for these and the like Operations.

This is a Paradox indeed, that some Altitudes of the Sun, taken off the Meridian, should, without a Watch or Clock, enable us to know the Sun's Meridian Altitude: But the Thing is plain, and the Practise easy, and not above the meanest Comprehension.

The great Number of Ships and other Vessels, and of Mens Lives also, that may be saved by these very Methods, in twenty or a hundred Years, shall be a Witness before Heaven and Earth, that will speak for me and my Executors, in case I alone should remain a Loser, whilst Providence makes use of me, to procure so much Good to others.

INTRODUCTION.



SOON after Pendulum Watches and Clocks were invented, some Mathematicians did perceive that the Latitude might be found at Land, by observing two convenient Altitudes of the Sun, or of the Sun and a Star, or of two Stars, together with the Time between both Observations. I think I remember to have seen in the Works of Sir Jonas More an Idea of this Problem; and that, because of the Usefulness of it, and for other Reasons, he printed a Table of the Verfed Sines both Natural and Logarithmic, in order to facilitate the Trigonometrical Calculation, which the Solution of this Problem and of some others also requires.

But the Trigonometrical Rule which he gives for these Calculations, being, as I suppose, the very same which Mr. Sherwin gives in his Mathematical Tables, among which he has published also the Table of the Natural and Logarithmic Verfed Sines; and that Rule being clogged by those Authors, and not given in its Natural Simplicity, nor very easy to be remembered; we do not perceive that hitherto it has been made use of for the Advancement and Perfection of Geography, and much less for the Perfection of Navigation. Nay, describing the Uses of that Trigonometrical Rule, Mr. Sherwin expresses himself only in the following Words:

“This Proposition is of great Use to calculate the Distance of Places on Earth, according to the Arch of a great Circle, by their Longitude and Latitude given; the Distances of Stars, by having their Declinations and Right Ascensions, or Longitudes and Latitudes given: By means whereof the Altitudes of two Stars, or of the Sun, with the Difference of Time or Azimuth being observ’d at any Time off the Meridian, the Latitude may be found”. In these Words there appear no footsteps of any Pretensions, and much less of any Method to find by these Data the Latitude of Ships under Sail; whereas I shall give a full Solution of this most useful Problem, though a Ship should alter its Latitude even by a hundred Miles or more, between both Observations. And I shall confirm that Solution by a notable former Experience, which may soon be followed by many more.

N. B. The Intricacy of the said Trigonometrical Rule printed by Mr. Sherwin is such, that in the two Examples given by him he has committed several Mistakes. In the use of that Rule he must write down eight Lines of Calculation, whereas I write but six. And besides he must take the half of a given Arc, and write down the double of a Logarithmic Sine, instead of my writing the Numbers as they are actually given, or else as they are found in the Tables. By which means I spare much Trouble, and am less exposed to Numerical Mistakes, and have sooner done; which may be sometimes of the utmost Importance at Sea.

However the said Trigonometrical Rule agrees in effect, by a round-about-way, with the Rule which I found a great many Years ago: Which last Rule makes part of the Latin Manuscript written by me in 1716, for the Improvement of Navigation; and is as follows. Neither did I perceive the near Agreement of those two Rules, till the end of the Year 1726.

TRIGONOMETRICAL PROPOSITION.

In this Projection of the Sphere ADPEFA, Fig. 1. which supposes the Eye at an infinite Distance, let APB be any Spherical Triangle proposed. Take the Arcs PD, PE, equal each of them to the Side PB. Draw the Chord DBE. Draw DG parallel to the Diameter AHICF; and draw the Lines DH, BGI perpendicular to the same.

The Radius | is to the Verfed Sine of the Angle APB || as the Sine of DP or of PB | is to DB.

So then $DB = \frac{\text{Verf. Sin. of the Angle APB} \times \text{Sine of DP}}{\text{Rad.}}$

Rad. AC | is to the Sine of AP || as DB found above | is to DG or HI = $\frac{\text{Sin. AP} \times \text{Sin. DP} \times \text{Verf. Sin. APB}}{\text{Rad. quadr.}}$

From this last Analogy I derive the following Rule.

Two Sides of a Spherical Triangle being given, and the Angle between them, to find the third Side.

SOLUTION.

To the Logarithm of the Verfed Sine of the given Angle, add the two Logarithmic Sines of the two Sides; and from the Sum cut off twice the Radius. Then seek the Natural Number of the remaining Logarithm. (That Number will give you the Length of HI.) To which Natural Number add the Natural Verfed Sine (AH) of the Difference of the Sides. The Sum will give the Natural Verfed Sine (AI) of the third Side or of the Base (AB.)

By the same Rule, Having the Three Sides of a Spherical Triangle, you may find the Angle opposed to that Side which you chuse to call your Base.

If for any Angle, for instance the greater Angle, and for its opposite Side, we take their Complements to 180 Degrees; the Angles shall be turn’d into Sides, and the Sides into Angles. And by consequence the Trigonometrical Proposition here demonstrated will serve for the Two following Cases also.

In a Spherical Triangle, Two Angles being given, and the Side between them; to find the third Angle.

And likewise, Three Angles being given, to find the Side opposite to which Angle you please.

If a Natural Verfed Sine exceed the Radius; from the same cut off the Radius, and the Remainder will be the Natural Sine of the Excess of the Angle or Arc above 90 Degrees. If an Arc exceed 90°, its Natural Verf. Sine is the Radius added to the Natural Sine of the Excess of the Arc above 90°.

A Logarithmic Verfed Sine that exceeds the Radius answers to a Natural Number, from which if you cut off the Radius, the Remainder is the Natural Sine of the Excess of the Arc or Angle above 90 Degrees.

If you continue the Side AB to a Semicircle ABF, you may chuse to resolve either the Triangle APB, or in its stead the Triangle PBF; for the Solution of the one involves the Solution of the other. Therefore chuse that which may be more easily solved.


There is a considerable Advantage in Trigonometrical Calculations, if one uses Natural Verfed Sines, and their Logarithms, rather than Natural Sines and their Logarithms. But of this I may speak elsewhere.

PROBLEM.

To find the Latitude, by Two Altitudes of the Sun and the Time between them; supposing the Observations to have been made from the same Place, or from different Places, whether at Sea or Land. And likewise to find the Hours, and the Sun's Azimuths, at the Times of both Observations.

SOLUTION.

EXAMPLE I.

Fig. 2.  SUPPOSE that in 1727. Nov. 3. O.S. in the Afternoon, the Sun being in A, Fig. II. its Distance AP from the North Pole P being $108^{\circ}13'$, you observed the Distance AZ, from the Zenith Z, after allowing for the Refraction occasioned by the Atmosphere, to have been $75^{\circ}14'$ when your Watch or Clock did shew $2^h 25^m 22^s$.

And that Nov. 7. in the Morning, the Sun being in B, its Distance BP from the North Pole P being $109^{\circ}12'$, you observed likewise its Distance BZ from the Zenith to have been $77^{\circ}41'$ clear of Refraction, when the Hour by the Watches was such that, upon the whole, you conclude the Angle APB, contained between the Sun's Horary Circles PA and PB, did answer to $5^h 21^m 20^s$ of Time, or to $80^{\circ}20'$. And this must be after allowing for the Equation $42''$ of Time, being so much as a Watch fitted to the Sun's mean Motion ought then to go slower than the Sun in $3\frac{1}{2}$ Days; and after allowing also proportionably for what you know or do suppose your Watches did get or lose every Day upon the Sun's mean Motion.

I speak here of Watches in the Plural Number, because it will be safest at Sea to have at least three of them. Major Holmes had three Watches, when he made those notable Experiments related in the Philosophical Transactions. If you have but two, the one may have some Irregularities, which could not be discovered so well by its differing from a second Watch. But the concurring Witness of two or more Watches against it, discovers better the Error, or at least gives the Hour surer, by taking a Medium between them all.

Wherefore these four short Operations, all perform'd by our single and most easy Rule, would give you the Latitude of the Place $48^{\circ}53'$. if the Observations were made by Land, or the Ship had remain'd unmoveable, or was return'd to the same Place again. It is to be hoped that the great Facility of this Method will invite many Travellers by Land, to take this obvious means of rectifying our too defective Geography.

V. To find the Angle BPZ, and by consequence the Angle APZ also, or the Hours answering those Angles, work thus:

Qu. Sin. Verf. BPZ?	941087	Sin. Verf.	42. $3\frac{1}{2}$	= APB
Sin. PB,	109.12	997515		
Sin. PZ,	41.7	981796		
		920398	L. of 15995	
Sin. Verf. Diff. 68. 5	(9 Char.)	62674		
Nat. Verf. Sin. BZ, $77^{\circ}41'$		78669		
			38.16 $\frac{1}{2}$	= APZ answering $2^h 33^m 6^s$ after Noon.

Through the Point Z draw the Circles or Arcs ZS and ZQ, whose Poles are A and B.

The same Rule by which those five preceeding Operations were made, may serve likewise to find the Azimuthal Angles PZA, PZB; especially if any of those Angles at Z be so far uncertain, that you are not sure whether it be bigger or smaller than a Right Angle. Otherwise you may use the Proportion between the Sines of the Sides and the Sines of their opposite Angles, as I do here; the Calculation being a little shorter.

VII. To find PZA.

Sin. AZ,	75.14	0.01459	Comp. Ar.
Sin. APZ,	38.16 $\frac{1}{2}$	9.79200	
Sin. AP,	108.13	9.97767	
Sin.	37.29	9.78426	
Suppl.	142.31	= PZA. PZB = $52^{\circ}31'$.	

I. In the Triangle APB, I find the Side AB by my Rule demonstrated heretofore; which I do as follows:

Log. Verf. Sin. APB	80.20.	992017	
Sinus PB	109.12.	997515	
Sinus PA	108.13.	997767	
		2987299	Log. Num: 74643
Nat. Verf. Sin. of the Dif. of	(6. being Chara-}		
the Sides	0.59.5	sterick	15
Nat. Verf. Sin. of AB =	$75.19\frac{1}{2}$		74658

II. The Angle PBA might be found by the Sines of the Angles being proportional to the Sines of their opposite Sides: But this Proportion leaving you to seek whether the Angle sought be greater or smaller than 90° , and therefore exposing you sometimes to mistake the Supplement to 180° instead of the Angle it self, it will be safest to use the preceeding Rule; and this must be done as follows.

Q. the V ^l . Sin. of PBA?	1009724	{ L. of 125093 which is the V.S. of 104°.
Sin. PB 109.12.	997515	
Sin. BA 75.19 $\frac{1}{2}$.	998559	
	Log. 1005798	<div style="border: 1px solid black; padding: 2px; display: inline-block;"> $v_c = \frac{v_c - v_x}{\delta_{A \& B}}$ </div>
Nat. Verf. Sin. of the Dif. of }		114283
the Sides 33.52 $\frac{1}{2}$ }	(9 Char.)	
Nat. Ver. Sin. of PA=Rad. + Na. Si. of 18°.13'		16978
		= 131261

III. The Angle ABZ is found thus, and also the Angle PBZ.

Qu. the Ver ^l . Sin. of ABZ?	989625	V. Si. of 77.44=ABZ
Sin. B Z	77.41	998989
Sin. A B	75.19 $\frac{1}{2}$	998559
		<u>104.22=ABP</u>
		26.48=PBZ.
	987173	74427
Na. Ver. Si. of the Dif. 2.21 $\frac{1}{4}$ (6 Char.		<u>85</u>
Nat. Ver. Sin. of AZ 75.14 (9 Char.		74512

IV. Now to find PZ work thus.

Si. Ver. PBZ,	26.48'	903106	
Sin. PB,	109.12	997515	
Sin. BZ,	77.41	998989	
		8.99610	99106
N.S.V. Dif.	31.31	(9 Char.)	14751
		24662	Si. V. PZ = $41^{\circ}7'$.
So then the Latitude of the Point Z is			48.53.

VI. To find PZB.

Sin. BZ,	77.41.	0.01011	Comp. Ar.
Sin. BPZ,	42. $3\frac{1}{2}$	9.82600	
Sin. PB,	109.12.	9.97515	
Sin.	40.21	9.81126	
		139.39	Suppl. = PZB. PZQ = 49.39 .

VIII. In order to verify this whole Calculation, I find the Angle AZB by our former Rule.

Quar. A Z B?	9.89726	Si. Ver.	$77^{\circ}50' = A Z B.$
Sin. B Z, 77.41	9.98989		$139.39 = P Z B.$
Sin. A Z, 75.14	9.98541		$142.31 = P Z A.$
	9.87256	74569	
Si. V. Na. 2.27	6 Char.	91	
Si. V. Na. A B, 75.19 $\frac{1}{2}$		74660	

360. 0 This Sum confirms
the whole Calculation
to be right.

This Sum confirms the whole Calculation to be right.

Thus

Thus those eight Calculations may be perform'd by the Help of our single Rule: To which you may intermix if you please, as I have done, that most easy but equivocal Rule, that the Sines of the Sides are proportional to the Sines of their opposite Angles. For this last Rule may serve to make the II. V. VI. VII. and VIII. Calculations. You shall see hereafter the great Use of this Preparation, even for finding the Latitude at Sea.

If two Persons that understand this Method help one another in their Calculation, the one seeking the Numbers in the Table while the other does write, the Work may be done in half the time, and with more Certainty; since the one will prevent the Mistakes which the other by Inadvertency might fall into. Many other Abridgments might be made use of, both for Ease and Diligence. For instance, some Schemes may be printed, or else engraven upon a Slate, with the whole Process of the Calculation, wanting only the particular Numbers to be inserted each in its proper Place: For thus the trouble of so much thinking, and writing, and verifying, and of making a new Scheme, will be spared; and the Danger of falling into Mistakes will be very much diminished. Now in some ticklish Circumstances of many sorts, it may be of the utmost Consequence at Sea to gain Time, to alter as soon as may be the Course of a Ship, or even of a Fleet.

EXAMPLE II.

Suppose, now, that, having made at Sea the very same Observations of the Sun, and at the very same Days and Hours, as in the first Example, your Ship, in the mean time between the Observations, did sail almost a South South East Course, so that you conclude, upon the whole, its Course did amount to an Arc of a great Circle of 156 Minutes or Miles in length, and did make at first an Angle of 159° with the Meridian.

It is asked which were both your Latitudes, at the Times of the Observations.

Which were the true Hours of the Day, at the Times and Places of both Observations.

And what Difference of Longitude results between both Places of Observations.

This Example as well as Example III is offered here with great Disadvantage: For, here is no mention of any the like antient Observations, by which the Latitude of the Point S (Fig. III) might be better ascertained. The Interval of Time between the Observations is very great; and so is likewise the Run of the Ship between the Observations. Lastly, the Arcs SZ and ZQ cut each of them the Meridian PZ with a great Obliquity; whereas it would be much better if one of those Arcs, and especially the Arc ZQ as belonging to the Second Observation, did cut the Meridian as near as might be at Right Angles.

When so many Difficulties occur, a ready Calculator will not grudge beginning his Calculation over again, after having made such Alteration in his uncertain Data, as his Conclusions will best direct him to. But he must be so much the more careful in procuring an Altitude nearer to the Meridian as soon as may be; or at least two Altitudes not separated by so long an Interval of Time.

To resolve this useful Problem I take the following Steps.

I. I make the very same Suppositions, and the very same seven or eight Calculations as in the first Example, in order to find not only the Dormant or Borrowed Zenith Z, which would have resulted from the Observations, if the Observer had been at rest; but likewise all the Arcs and Angles which are given or found by those very Suppositions and Calculations.

II. By this means, supposing the little Triangle XZY (in the III Figure) to result from those Calculations, and to be a Rectilinear Triangle, since it differs but little from one; and supposing also the Meridians passing thro' the several Points X, Y and Z to be sensibly parallel; and $XY = 156$ Minutes or Miles to represent your Course: You have all the Angles in the Triangle XZY, namely, the Angle at X of $51^\circ 31'$; the Angle at Z of $77^\circ 50'$; and the Angle at Y of $70^\circ 39'$.

Thus calculating the Side ZX, or else construing the Figure it self, you will have with no great Error the Point X for the Place of your Ship, at the Time of the first Observation.

III. This however is but a very near Approximation, which you may correct either by a mechanical Construction, or rather by a further Calculation, as follows; or, if you will, by both.

As your Approximation may best direct you, take two different Values of ZX or Zs; the Difference between them being ten Minutes, or, if you will, twenty Minutes: And for these two Values find severally in the Triangles PsZ their Sides Ps.

IV. Then, for the two Values of Ps, find in the same Triangles PsZ their Angles sPZ.

V. Then, for the two Values of Ps, find, in the Triangles Psq, the two Values of Pq.

VI. Then in the same Triangles Psq find the two Values of the two Angles sPq. And so you will have the two Angles qPZ likewise.

VII. Then, in the two Triangles qPB, find the two Values of qB.

VIII. Now QB being given, as here of $77^\circ 41'$, its Differences from both the Values of qB will be known. And if you work proportionably on the respective Differences of both the Values of Ps, sPZ, Pq, sPq and qPZ, you will find the true corrected Values of PS, sPZ, PQ, SPQ and QPZ; and by consequence you will have the full Resolution of the present Problem.

I. All the Work of the first Step or Article is already dispatch'd in the first Example.

II. The Side ZX in the Triangle ZXY will be found by this Proportion.

$$\begin{array}{l} \text{Sin. } 77.50 \mid \text{Sin. } 70.39 \parallel 156 \text{ Miles} \mid \begin{cases} 150.57 \text{ Miles} = ZX \\ = 2^\circ 30' 34'' \end{cases} \\ \text{Comp. Ar. Sin. } 77.50. \quad 0.00987 \\ \text{Sin. } 70.39. \quad 9.97475 \\ \text{Log. } 156. \quad 2.19312 \\ \hline \quad \quad \quad 2.17774 \text{ Log. } 150.57. \end{array}$$

III. Let the two Values of Zs be taken, the one $= 2^\circ 22'$.

So then, in the Triangles PsZ, I find the Sides Ps as follows.

$$\begin{array}{l} \text{Sin. Verf. PZs, } 52.31 \quad 959270 \\ \text{Sin. PZ, } 41.7 \quad 981796 \\ \text{Sin. sZ, } 2.22 \quad 861589 \end{array}$$

$$\begin{array}{l} 802655 \text{ Log. } 1063 \\ \text{Si. Ve. Na. Dif. } 38.45 \text{ (9 Ch.)} \quad 23012 \end{array}$$

$$\begin{array}{l} 23075 \quad 39^\circ 43' = \text{Ps.} \\ \text{PS} = 39.39 \end{array}$$

And the other Value of Zs $= 2^\circ 32'$.

$$\begin{array}{l} \text{Sin. Verf. PZs, } 52.31 \quad 959270 \\ \text{Sin. PZ, } 41.7 \quad 981796 \\ \text{Sin. sZ, } 2.32 \quad 864543 \end{array}$$

$$\begin{array}{l} 805609 \text{ Log. } 1138 \\ 38.35 \text{ (9 Ch.)} \quad 21830 \end{array}$$

$$22968 \quad 39^\circ 37' = \text{Ps.}$$

IV. To find the Value of the first Angles PsZ and sPZ .

$$\text{Sin. } Ps | \text{S. } sZP || \text{Sin. } sZ = 2.22 | \text{Sin. } sPZ = 2.56 \frac{2}{5}.$$

$$39.43 | 52.31 || \text{Sin. } PZ = 41.7 | \text{Sin. } PsZ = 125.15:$$

$$\text{Com. ar. Sin. } 39.43.0.19450 \quad \text{Comp. ar. Sin. } 39.43.0.19450$$

$$\text{Sin. } 52.31.9.89956 \quad \text{Sin. } 52.31.9.89956$$

$$\text{Sin. } 41.7.9.81796 \quad \text{Sin. } 2.22.8.61589$$

$$\text{Sin. } 54.45.9.91202 \quad \text{Sin. } 2.56 \frac{2}{5} 8.70995$$

$$PsZ = 125.15$$

$$SPZ = 3^{\circ} 4' \frac{2}{5}$$

To find the Value of the second Angles PsZ and sPZ .

$$\text{Sin. } Ps | \text{Sin. } sZP || \text{Sin. } sZ = 2.32 | \text{Sin. } sPZ = 3.9 \frac{1}{5}$$

$$39.37 | 52.31 || \text{Sin. } PZ = 41.7 | \text{Sin. } PsZ = 125.5$$

$$\text{Co. ar. S. } 39.37.0.19542 \quad 39.37.0.19542$$

$$\text{S. } 52.31.9.89956 \quad 52.31.9.89956$$

$$\text{S. } 2.32 \frac{8.64543}{8.74041} \quad \text{Sin. } 41.7.9.81796$$

$$9.91294 \quad 8.54.55 \frac{2}{5}$$

$$PsZ = 125.55$$

$$Psq = 159.$$

$$Zsq = 33.55$$

V. To find Pq in the first Triangle Psq .

$$\text{Sin. Verf. } 159^{\circ} \quad 1028636.19.3358$$

$$\text{Sin. } Ps \quad 39.43.980550$$

$$\text{Sin. } sq \quad 2.36.865670$$

$$874856 \quad 56048$$

$$\text{Sin. Verf. Nat. } 37.7(9) \quad \frac{20259}{25864}$$

$$\text{Sin. Verf. } Pq = 42.9 \quad 25864$$

$$-3 \frac{1}{5}$$

$$PQ = 42.5 \frac{2}{5}$$

To find Pq in the second Triangle Psq .

$$\text{Sin. Verf. } 159^{\circ} \quad 1028636$$

$$\text{Sin. } Ps \quad 39.37.980458$$

$$\text{Sin. } sq \quad 2.36.865670$$

$$874764 \quad 55929$$

$$\text{Sin. Verf. Nat. } 37.1(9) \quad \frac{20154}{25747}$$

$$\text{Sin. Verf. } Pq = 42.3 \frac{1}{5} \quad 25747$$

VI. To find the Angle sPq in the first Triangle Psq .

$$\text{Sin. } Pq, 42^{\circ}.9' | \text{Sin. } Psq, 159^{\circ} || \text{Sin. } sq, 2^{\circ}.36' | \text{Sin. } sPq.$$

$$\text{Sin. } 42.9.0.17323$$

$$\text{Sin. } 21.9.55433$$

$$\text{Sin. } 2.36.8.65670$$

$$2.56 \frac{2}{5} = sPZ$$

$$8.38426 \text{ Sin. } 1.23 \frac{1}{5} = sPq$$

$$1.23 \frac{1}{5} = SPQ$$

$$1.33 \frac{4}{5} = qPZ$$

$$\text{But } ZPB \text{ was found } 42.3 \frac{1}{5} = ZPB$$

$$43.36 \frac{2}{5} = qPB$$

$$43.44 \frac{1}{5} = QPB$$

$$7 \frac{3}{5} \text{ Correction.}$$

To find the Angle sPq in the second Triangle Psq .

$$\text{Sin. } Pq, 42.3 \frac{1}{5} | \text{Sin. } Psq, 159^{\circ} || \text{Sin. } sq, 2^{\circ}.36' | \text{Sin. } sPq.$$

$$\text{Sin. } 42.3 \frac{1}{5} 0.17386$$

$$9.55433$$

$$8.65670$$

$$3.9 \frac{1}{5} = sPZ$$

$$8.38489 \text{ Sin. } 1.23 \frac{1}{5} = sPq$$

$$1.45 \frac{4}{5} = qPZ$$

$$42.3 \frac{1}{5} = ZPB$$

$$43.49 \frac{1}{5} = qPB \text{ Diff. } 12.633 \frac{1}{5}$$

VII. To find qB in the first Triangle qPB .

$$\text{Sin. Verf. } 43.36 \frac{2}{5} 944085$$

$$\text{Sin. } PB, 109.12 997515$$

$$\text{Sin. } Pq, 42.9 982677$$

$$924277.17489$$

$$67.3 \quad 61007$$

$$qB = 77.35.78496. \text{ Instead of } 78668 \frac{2}{5},$$

$$\text{Which is the Natural Versed Sine of } QB = 77.41$$

To find qB in the second Triangle qPB .

$$\text{Sin. Verf. } 43.49 \frac{1}{5} 944476$$

$$\text{Sin. } 109.12 997515$$

$$\text{Sin. } Pq, 42.3 \frac{1}{5} 982595$$

$$924586$$

$$67.8 \frac{1}{5}$$

$$qB = 77.44 \frac{6}{5}$$

$$78778$$

$$78496$$

$$282$$

$$345$$

$$564$$

$$115$$

$$188$$

$$= 0.6117$$

In this Proportion
must all the Differen-
ces be corrected.

If from $APZ = 38^{\circ}.16 \frac{1}{5}$ you take $SPZ = 3^{\circ}.3 \frac{4}{5}$, you will have $APS = 35.12 \frac{2}{5}$ answering to the true Hour in S when the first Observation was made, viz. $2^h 20^m 51^s$ in the Afternoon.

The difference of Longitude between the Points S and Q is $1^{\circ} 23 \frac{1}{5}$ answering to $5^m 33^s$ of Time.

And the Angle $QPB = 43^{\circ} 44'$ answers to the true Hour in Q when the second Observation was made, viz. $2^h 54^m 56^s$. before it was Noon.

Now whereas in the Points S and Q you did set down the Hours by your Watches, at the Times of the first and the second Observations, which otherwise would be but lame; if these Hours given by the Watches be compar'd with the true Hours of the Sun as determined here, having a due regard to the Equation of the Sun, and to the Swiftnefs of your Watches in 24 Hours, and to the Places where your Watches were compared last with the Heavens: You will have all the Lights that these Observations can give you, in reference to the Longitude of the Points S and Q , and also to the Swiftnefs of your Watches, and the Length and Position of SQ .

EXAMPLE II, Reduced to a greater Simplicity.

If, rather than to be at the trouble of making so full a Calculation, you chuse to seek only your Latitudes, or else the Distances PS and PQ from the Pole, by an Approxi-

mation erring but a few Minutes at most; you may, from the Value of ZX ($N^{\circ} 2.$) calculate (instead of $N^{\circ} 3$) the Value of PX , which will be found $39^{\circ} 37'$ as follows, differing but $2 \frac{1}{2}$ Minutes from the corrected Value of PS .

$$\text{Sin. Verf. } PZX \quad 52.31. \quad 959270$$

$$\text{Sin. } 41.7. \quad 981796$$

$$\text{Sin. } 2.34.5.865133$$

$$806199 \text{ Log. } 11534$$

$$38.32.55$$

$$21810$$

$$22963.39^{\circ}.37' = PX.$$

Then to find the Angle PXZ , you may say;

$$\text{Sin. } PX | \text{Sin. } PZX || \text{Sin. } PZ | \text{Sin. } PXZ = (54.55)$$

$$39.37 \quad 52.31 \quad 41.7$$

$$125.5$$

$$\text{Co. ar. Sin. } 39.37 \quad 0.19542$$

$$159. = PXq$$

$$52.31 \quad 989956$$

$$33.55 = ZXq$$

$$41.7 \quad 981796$$

$$52.31 = PZX$$

$$9.91294$$

$$18.36 \text{ Inclination of } Xq \text{ to the Meridian } PZ.$$

And so having the Inclination of Xq to the Right Line ZX , you will find the Difference of Latitude between X and q as follows.

[Rad | Cofin. 18.36 || 156 Mi. = Xq | 147.85 = 2°.27'.51" Difference of Latitudes.

Cofin. 18.36. 9 97670

Log. 156 2 19312

2.16982 Log. 147.85.

39.37.

42.4.51"

42.5.24

0.0.33"

Comes for Pq, instead of

Found by the Calculation at large for PQ.

Error.

And this degree of Exactness is undoubtedly sufficient.]

Or else you may say

Sin. 77.50 | 156 || Sin. ZXq | Zq = 89.275.

33.55

77.50. 0.01099

156. 2.19312

33.55. 9.74662

1.95073

Cof. 49.39. 9.81121

1.76194

Log. 89.275 = Zq.

Log. 57.802

Rad | Cof. 49.39 || Zq | Diff. of Lat. between Z and q. 57.48"

41.7.

Pq = 42.4.48

PQ = 42.5.24

Error c. 0.36

EXAMPLE. III.

Suppose now that having made at Sea the very same Observations of the Sun as in the first Example, your Ship, in the mean time between the Observations, did sail by a Loxodromic Course, 233.09 Miles in length, making with the Meridian an Angle of 59°; steering thus almost North East and by East, along the Line SQ, Fig. II. It is required to find the Latitudes of the Points S and Q where your Ship was at the Times of the Observations; and likewise the Difference of Longitude between those two Points; and the true Hours of the Sun, or the Angles APS, BPQ when the Observations were made.

To resolve this useful Problem, and the others like it, we may take the following Steps. But here if I give an Example wherein I suppose so long a Run as 233.09 Miles, and so long an Interval between the Observations as 34 Days, I hope it will appear there is no Presumption in this, since I shall make a right Use of it: And I am satisfied that Sea Jewel-Watches or Clocks may be made or rendered still more accurate for Naval Uses, than they may now seem to be; and that there are Methods for making much accurater Observations at Sea, and a surer Estimation of the swiftness of a Ship, than could be procured by any ordinary Way. Besides, this Example is taken from Nature. For it will but too often happen that Navigators will have no better Observations to work-upon, especially in Winter-time; and will think themselves happy, if they can but have them, and know what to do with them, but chiefly when they can have their second Observation not too far from Noon.

I. The first Step to resolve this Problem, is to make the same Suppositions, and the very same seven or eight Calculations as in the first Example, in order to find not only the dormant or borrowed Zenith Z, which would have resulted from the Observations if the Observer had been at rest; but likewise all the Arcs and Angles which are given or found by those very Suppositions and Calculations. To which may be added the finding of the Difference of Latitude between the Places of the Ship in the first and in the second Observation.

II. By this means, supposing the little Triangle SZQ Fig. II. and V. to be a Rectilinear Triangle, since it differs but little from one; and supposing that the Meridian PZ cuts the Line SQ in N: You will have all the Angles in the Triangles SNZ and ZNQ, and by consequence in the Triangle SQZ also. By which means, having the Side SQ, you will find with no great Error, the Sides SZ, ZQ, and the Distances of the Parallels of Latitude passing through S and Q, from the Pole P or else from the Equator.

III. This however is but a near Approximation, which you may correct either by a mechanical Construction, or rather by a further Calculation as follows, or if you will by both.

As your Approximation may best direct you, Take either two different Values of PS, or else three different Values of PS in an Arithmetical Progression, the common Difference between the Terms of that Progression being ten Minutes, or, if you will, twenty Minutes.

Then for the least and the biggest of those Values of PS calculate at once, by the Rule given in *Philos. Trans.* N° 219, or in *Sherwin's Mathematical Tables*, the resulting Sides PQ, and likewise the resulting Angles SPQ.

IV. Afterwards for the two or three assumed Values of PS calculate at once the resulting Angles APS. Add them to their proper Angles SPQ. And thus the Angles QPB becoming known, find at once the resulting two or three Sides BQ, which will not much differ from the given Side BZ. When I say that these two or three sets of Calculations must severally be made at once, I mean only that you will by these Means spare a part of the Time and Trouble in calculating. For the Tables will present to your view at once two or three of the Elements or Numbers which must be employ'd in your Calculations.

V. These Preparations being once made, you will easily determine the true Numbers resulting for the Complements of your Latitudes at the Times of your Observations, or for PS and PQ; and likewise for the Angles APS, SPQ and QPB, or for the true Hours when you made your Observations, and for the Difference of Longitude between the Places you were in at those very Times, as far as you may depend upon the Truth of your Observations.

I. Now then to proceed in our Example by the Steps set down here; You have already, in your seven or eight former Calculations, got over the first Step. To which, as an Appendix or a further Preparation, may be added here the finding of the Difference of Latitude between the Points S and Q. For by the Nature of the Loxodromic Line, As the Radius, is to the Cofine of the Angle of your Course with the Meridian; so is SQ, or 233.09 Miles, to the Difference of Latitude between your Ship in S and your Ship in Q: Which Difference is found here to be 2° 3'.

Cofin. of 59° 97.184.

Log. 233.09 2.36752

207936 Log. of 120.05 Miles = 2° 3'.

II. Sin. SZQ = 102°.10' | is to SQ = 233.09 || Sin. SQZ = 9° 21' | SZ = 38'.739
 || Sin. ZSQ = 68.29 | ZQ = 221'.83
 Com. Ar. Sin. 77.50. 0.00987
 Sin 9.21. 9.21076
 Log. 233.09 2.36752
 Log. SZ = 38'.739 1.58815
 Log. ZQ = 221'.83 2.34602
 Sin. 68.29. 9.96863

These Calculations serve to find, by anear Approximation, SZ, ZQ; ZL and ZL: And by consequence to find the Latitude of the Points S and Q pretty near the Truth.

R | Cofine 52°.31' || SZ | ZL.
 R | Cofine 49°.39' || ZQ | ZL.
 Cofine 52.31. 9.78428
 Log. SZ 1.58815
 1.37243 L. ZL = 23'.574 = 23'.
 PZ = 41.7
 ZL = 23'.
 PS = PL = 40.43'.
 Cof. 49.39. 9.81121
 L. ZQ 2.34602
 2.15723 L. ZL = 143'.62 = 2° 23'.
 PZ = 41.7
 ZL = 2.23'.
 PQ = PL = 38.43'.
 So then the Difference of Latitudes L.L which ought to be 2° 3' comes forth only 2°. Which however intimates a pretty near Approximation.

III. I take then for the three different Values of PS, successively, 40°.30', and 40°.40', and 40°.50'. And by the first and last of them I find the Angles SPQ as follows:

PS = 40.30 } 20.15 Tang. 956693
 L.L = 2.3 }
 PQ = 38.27 } 19.13' 954248
 244.5 Log. 2.38614
 Log. Tang. 59°. 10.22123
 12.60737
 Constant Log. 10.10151
 Log. 320'.52 2.50586

So then SPQ = 5° 20'.5

PS = 40.50 } 20.25 Tang. 957081
 2.3 }
 PQ = 38.47 } 19.23' 954653
 242.8 Log. 2.38525
 10.22123
 12.60648
 10.10151
 Log. 319.87 2.50497
 So then SPQ = 5° 20'.

Then I find the Angles APS and QPB as follows:

Sin. Verf. APS? 930430 APS = 37.1
 Sin. AP, 108.13 997767 SPQ = 5.20'
 Sin. PS, 40.30 981254 QPB = 37.58'
 909451 12431 80.20
 Sin. Verf. Nat. 67.43 62081
 Sin. Verf. Nat. 75.14 74512
 Sin. Verf. APS? 931211 APS = 37.22
 Sin. AP, 108.13 997767 SPQ = 5.20'
 Sin. PS 40.40 981402 QPB = 37.37'
 910380 12700 80.20
 67.33 61812
 75.14 74512
 Sin. Verf. APS? 931971 APS = 37.42
 Sin. AP, 108.13 997767 SPQ = 5.20
 Sin. PS 40.50 981549 QPB = 37.18
 911287 12968 80.20
 67.23 61544
 75.14 74512

And here I find the Sides BQ as follows, in the Triangles QPB.

Sin. Verf. QPB, 37.58' 932576
 Sin. PB, 109.12 997515
 Sin. PQ, 38.27 979367
 909458 12433
 Sin. Verf. Nat. 70.45 67031
 Sin. Verf. Nat. BQ 79464 78.9 = BQ
 Sin. Verf. QPB, 37.37' 931811
 Sin. PB, 109.12 997515
 Sin. PQ, 38.37 979526
 908852 12261
 Sin. Verf. Nat. 70.35 66756
 Sin. Verf. Nat. BQ 79017 77.53 = BQ
 Sin. Verf. QPB, 37.18 931075
 Sin. PB, 109.12 997515
 Sin. PQ, 38.47 979684
 908274 12099
 Sin. Verf. Nat. 70.25 66482
 Sin. Verf. Nat. BQ 78581 77.38 = BQ

Now if BQ be made of 77° 41' as it was observed, you will have proportionably, by the preceeding Calculations; QPB = 37° 22'; which answers to 2^h 29^m 12^{sec}. And this gives the Hour when your last Observation was made, viz. 9^h 30' 48" in the Morning.

SPQ = 5° 20'; which gives your Departure or Change of Longitude.

APS = 37° 38'; which answers to 2^h 30^m 32^{sec}. for the Time of the Day when your first Observation was made in the Afternoon.

PS = 40° 48'. And by consequence 49° 12' will be your Latitude in the first Observation.

PQ = 38° 45'. And by consequence 51° 15' comes forth for your Latitude in the second Observation.

Notwithstanding the length of your Way, that is of SQ, and your great Distance from the Equator; you might nevertheless have spared the two middlemost Calculations, which here only justify that the Variations of BQ and of the Angle APS are gradual and very regular.

And

And here, by an Inspection of a Figure regularly made, and fitted to your Example, you are hitherto left to judge what Uncertainties you may be at, whether in reference to your Watches, or to the Length and Position of your Way; and how much those Uncertainties may affect either your Latitudes, or your Difference of Longitude between the Places of your Ship in S and Q. But as all these things have their Bounds, which it has been my Application for these many Years still to reduce within a small Compass, so we can be, especially with those Helps I would make use of, but a little mistaken in reference to the Latitude, though we may be more mistaken in reference to the Longitude.

But the sequel of your Observations will further help to set you right, as to those remaining small Doubts or Errors; whether you can come to a Meridian Observation; or only come to an Observation not very different from Noon; or else procure two proper Observations within a shorter Distance of Time from one another, suppose within less than one Day: Which are the means of avoiding, as to the Latitude, the greater Errors that might arise from the Irregularities of your Watches or Clocks.

EXAMPLE IV.

Suppose, for instance, that on the seventh of November you took, among the Clouds, a second Observation of the Sun, when it was come to the Point *a*, Fig. IV. and found its true Distance *az*, from *z* the new setting Zenith $70^{\circ} 5'$, or else $70^{\circ} 35'$, just $2^h 25^m 14^s$, after the other Observation was made in the Morning: And that, having had no Opportunity to follow the Sun any longer, you could pronounce nothing about your nearness to or remoteness from a Meridian Observation. For it is to be noted; that, because of the former *Data* in the preceding Observations, I may here chuse *az* in Conformity or in Disagreement to the same. The Conformity would be greater if I did chuse $az = 70^{\circ}.33'$ or there about. But the very Disagreement in the result of the Calculations will be more instructive, and give room to some Reflexions, which it will be proper to lay before you.

Therefore drawing the new Meridian *Pz*, you will have, by reducing the Time, viz. $2^h 25^m 14^s$, into Degrees, the Angle $aPB = 36^{\circ}.18'.30''$. $PB = 109^{\circ}.12'$, as before. $Pa = 109^{\circ}.13'$.

And you will find $Ba = 34.13\frac{1}{2}$ as follows:

I. Sin. Verf. BP a	=	36.18.30	928816
Sin. BP		109.12	997515
Sin. Pa		109.13	997510
			923841
Differ.		0.1	2 Char.
Sin. Verf. aB	=	34.13 $\frac{1}{2}$	17314 $\frac{1}{2}$

II. To find the Angle *PBa*, work thus.

Sin. 34.13 $\frac{1}{2}$		Sin. 36.18.30		Sin. 109.13		Sin. PB a.
Comp. Ar. Sin.		34.13 $\frac{1}{2}$		0.24994		
Sin.		36.18 $\frac{1}{2}$		9.77242		
Sin.		109.13		9.97510		
				9.99746		Sin. 83.49
						96.11 = PB a.

III. To find *aBz* and *PBz*.

Sin. Verf. aBz	984572.	72.36	aBz.
Sin. Bz	77.41	998989	23.35
Sin. Ba	34.13 $\frac{1}{2}$	975006	96.11
		958567	38519
Nat. Verf. Sin. 43.27 $\frac{1}{2}$			27416
Nat. Verf. Sin. 70.5 = az			65935

IV. To find *Pz* and consequently the Lat. also of the Point *z*

Sin. Verf.	23.35	8	92179
Sin. BP.	109.12	9	97515
Sin. Bz.	77.41	9	98989
		8.88683	7706
	31.31	9	Ch. 14751
Nat. Verf. Sin. Pz		22457	39 $^{\circ}$. 9' = Pz.
			50.51 Latit. of z.

V. To find *BPz* and *aPz*.

Sin. Verf. BPz?	933142.	38.14	BPz.
Sin. PB.	109.12	997515.	36.18 $\frac{1}{2}$
Sin. Pz	39.9	980027.	1.55 $\frac{1}{2}$
		910684	12789
Sin. Verf. Nat. 70.3			65880
Sin. Verf. Nat. Bz. 77.41			78669

VI. To find *PzB*.

Sin. Bz. 77.41	gives	0.01011
Sin.	38.14	BPz 9.79160
Sin. PB. 109.12		9.97515
		9.77686
		Sin. 36.44 $\frac{1}{2}$ +
		PzB = 143.15 $\frac{1}{2}$ -

VII. To find *Bza*.

Sin. Verf. Bza?	925269	34.48 $\frac{1}{2}$	= Bza.
Sin. Bz	77.41	998989	143.15 $\frac{1}{2}$
Sin. az	70.5	997322	178.4
		921580	16436
Sin. Verf. Nat. 7.36.7	Ch.		878
Sin. Verf. Nat. Ba, 34.13 $\frac{1}{2}$			17314

VIII. But to find likewise *Pza* directly.

Sin. az, 70.5	0.02678	Comp. ar.
Sin. aPz, 1.55 $\frac{1}{2}$	8.52622	
Sin. Pa, 109.13	9.97510	
	8.52810	Sin. 1.56
		or of 178.4 = Pza: Which coming forth as we did find it before, may serve for a Verification.

Thus the Decussations or Angles in *z* of the three great Circles, and, by consequence, of the two small Circles that fall here under consideration, being given, we may make a new Figure, in order to proceed by an Approximation.

Suppose then that your Ship, following still the same Course with an Inclination of 59° to the Meridian, did, between the two last Observations of the Sun in B and a,

Fig. IV. make a Run of 8.4 Miles or Minutes, represented Fig. VI. by QD: And your Numbers or Elements will result as in the Figure VI, where the Two small Circles aforesaid are QzK and zD.

For the preceding Calculations give you the Angles PzK or QzL, $53.15\frac{1}{2}$. KzD, $34.48\frac{1}{2}$. DzL, 91.56 . QzD, $145.11\frac{1}{2}$. zQD, $5.44\frac{1}{2}$. QDz, 29.4 .

So then resolving the Triangle QDz, you have these Analogies, and by them the Lengths of Qz and Dz.

Sin. 145.11 $\frac{1}{2}$ 8.4 Sin. 29.4 Qz = 7.15.		
34.48 $\frac{1}{2}$ 8.4 Sin. 5.44 $\frac{1}{2}$ Dz = 1.47.		
0.2436	0.2436	Pz = 39°. 9'
Log. 8.4 0.9243	0.9243	zL = 4.
Sin. 29.4 9.6865	Sin. 5.44 $\frac{1}{2}$ 8.9997	PL = PQ = 39.13 $\frac{1}{2}$
0.8544 Log. Qz = 7.15	0.1676 Log. Dz = 1.47.	PL = 39.8. 57.

Likewise by these other Analogies you will find zL and zL as follows.

Rad Cofine QzL = 53.15 $\frac{1}{2}$ Qz zL = 4.277	
Rad Cofine PzD = 88.4 Dz zL = 0.0496	
Log. Qz 0.8544	Log. Dz 0.1676
Col. 53.15 $\frac{1}{2}$ 9.7768	Col. 88.4 8.5281
0.6312 Log. zL = 4.277	2.6957 Log. zL = 0.0496 = 0.3"

Now PL or PQ which is here 39° 13 $\frac{1}{2}$, was found by the former Calculation 38° 45': Which Numbers differ by 28 $\frac{1}{2}$."

However, as the Latitude of the Point D is determined as exactly as it can be had, because of the Nearness of the Observation to the Time of Noon, we have little or nothing to change in it.

Neither can we change much the Latitude of the Point Q as determined here. For the Interval of Time being only 2^h 25 $\frac{1}{4}$, and QD being so short as 8.4 Miles; whether we consider the Uncertainty of its Length, or the Uncertainty of its Inclination to the Meridian, we can hardly suppose that Uncertainty to amount to 1 $\frac{1}{2}$ Minutes in the determination of the Latitude; which might reduce PQ to 39° 12'. The Uncertainty in the Altitudes may be a little greater: But, in the Practice, it will prove still more inconsiderable, if the Altitudes be taken by my Method. As to the Currents if there be any, I suppose you would do what lies in you to know them, that you might account for them, by increasing or diminishing proportionably the Length of your Course, and its Inclination to the Meridian.

The Uncertainty of the Time spent in describing the Line QD is likewise inconsiderable, both because of its being so short, and because an Error of some Minutes in that Time, would alter very little the Latitude of the Point D, that Point falling here so near the Meridian.

We must then refer the main of these Errors to the former Vth Figure, where we construed the Line SQ. If that Line SQ had been construed as the Result of two Observations made in the same Day, or made, the one in the Afternoon of one Day, and the other in the Morning of the next Day, and not so late as three or four Days after, as we did heretofore suppose: Then the Errors arising from any of the Uncertainties aforesaid would be so much the smaller, and would shew themselves such in your Calculation.

But here, as the Latitude of the Points D and Q Fig. VI. was ascertained by the subsequent Observation made in D, and this notwithstanding the great Length of SQ: So, on the other hand, the Latitude of the Point S ought to be ascertained by your Anterior Observation made before that other which answers to the Point S. For if the same Day that you made, or one or two Days before you made, the Observation S, you departed from a Harbour, or were in a Place, whose Latitude you know, or if you took then a Meridian Observation, or else if you took an Observation which fell not far from Noon; or lastly, if you took then any Observation to which you may compare the Observation made in S; Then you may, *mutatis mutandis*, know the Latitude of S with a sufficient Exactness, by the same Method as we have here determined the Latitude of Q. Likewise your Knowledge of the Latitude of the Point D gives you a good Foundation, upon which you may afterwards build your next Observations and Calculations.

Having these notable Helps, you will be enabled to pronounce more safely concerning the Longitude of the Points S, Q, D; especially comparing also the Hours of your Observations as found by your Watches or Clocks, with the Hours found by your Calculations. And in this

the best Watches will have the Advantage of those that are not so good: And my Sea Jewel-Watches will shew what may be expected from them; which I must leave to Experience. But this by the by: For at present what I aim at in this Discourse is chiefly the Latitude; on whose Knowledge our very Sailors make, in a great measure, to depend their imperfect Guesses at the Longitude.

As to the Angle QPD, which gives the Difference of Longitude between the Points Q and D, as resulting from the given Length of QD = 8.4: You need here take no further Trouble about it, that Line being so short, but only to make use of this Proportion sufficiently accurate for the Purpose, *viz.* SQ, is to the Angle SPQ; or 233.09 Miles, to 5° 20 $\frac{1}{2}$: as QD or 8.4 Miles, to the Angle QPD = 11' 34".

In the VIII Figure, suppose the Meridian Altitude was taken in O in the Parallel OE, three Days before the third of November; And that the same Day, *viz.* the 31st of October, in the Morning, the Sun's Altitude was likewise taken when the Ship was in H, and, if you will, in the Afternoon also, when the Ship was in K.

If your Ship did at first set out from C, describing in given Times the Lines CH, HK, KS, SQ, QD, whose Proportion is given, as far as your Estimation can approach the Truth; The very Proportions between those Lines, especially between OS and SQ, will give a great light for determining better the Position or Obliquity and Length of the Ship's Way; and the Difference of Longitude between O and D, or between C and D.

If, instead of Two Altitudes of the Sun, we have Two Altitudes of a Star, or of two Stars, or of the Sun and a Star, or even if we have One or Two Altitudes of some other Planets instead of the Sun, The same Method will serve to find the Latitude at Sea. Only it must be remembered that the diurnal Revolution of fixed Stars thro' 360 Degrees does amount, not to 24 Hours, but to 23^h 56^m 4^{sec}. The diurnal Revolution of other Planets may be derived from printed Ephemerides.

This however I have no design to treat of at present. Many others can do it at least as well as myself. And, before I publish what I have further in store for the Improvement of Navigation, I must see how that will be received which I now publish.

Thus you have here the Resolution of the Problem which we did propose. You have it, on one Side, as accurate as the nicest Calculators can desire. And as for them that chuse to work, as much as may be, by Figures and Instruments, you have here a Prospect of what may be done in that kind, whether they will work altogether by Figures, or rather work partly by Calculations also.

A successful Experiment of this Method was tried by me, in January 1713 $\frac{1}{4}$. We were coming from the Straights, aboard the Duke, Captain Nathanael Clerk Commander; and for several Days could take no Meridian Altitude. But one Day I promised him, from any Two convenient Altitudes of the Sun, taken among the Clouds, to find the Latitude: And accordingly, by Two Observations taken in the Morning, I did it, in perfect Agreement with what they did observe at Noon. He took a particular care that I should know nothing of the Meridian Altitude which they did find. And, in order to try

Fig. 8

my Method the better, he did even tell me, before I had gone through my Calculations, that we were in a Latitude different from ours by one whole Degree. But when I had done, I told him confidently, to his great Surprise, Captain, You said we were in such a Latitude: But I tell you we are in such another Latitude; naming those very Degrees and Minutes which himself and others had observed.

WAVES, in the open Seas, are exceeding regular, and may serve, as I have said, for a sort of Time-keeper. Their Number, as they pass under a Ship, may be continually mark'd of itself, by an Instrument which I call a *Kymometer*, and which will hereafter be of excellent Use for Navigation.

By this means, Two Ships being at first on the same Wave, and observing then the Numbers mark'd by their *Kymometers*, or causing them to agree, may know afterwards (from as far as they can hear and see any Signals given by One Ship to the Other, or any Signals given by a Third Ship placed in or near the Line that joins them both) How many Waves there is, at any time, between the Ships,

And when 1000 or 2000 or 3000 Waves, &c. have passed under the Ship, if a great Gun be fired once or several times, by one of the two Ships, or by the middlemost of the three Ships, at the Instant of their giving a Signal to the other or to both the other Ships; The Obliquity of the Waves to the Lines that join the Ships being observed also: Then the Length of the Time observed, between the Moments of firing the Guns and of hearing their Report, will be sufficient to calculate thereby the Breadth of each Wave.

And if one of the Ships remains in the same Place, or in the same unmoveable Line parallel to the Waves themselves, The Swiftnes of the Waves will be known also; and that so much the better, as the Line joining the Ships is less oblique to the Waves.

The Swiftnes of Sounds has been observed nicely by the late Mr. *Derham* R. S. S.

The Figure of Waves is very regular also. It seemed to me in the Ocean to represent the Line of Sines; where the Degrees of a Circle being reckoned on the Horizontal Abscissa perpendicular to the Waves, the Ordinates

perpendicular to the Horizon represent for each Degree their correspondent Sines.

And so, The very same Figure which the upper Part of the Wave has above the Natural Level of the Sea; the very same Figure, I say, has also sensibly, in an inverted Situation, the concave Part of the Wave, under the natural Level of the Sea:

In coming from the Straights in the Ship the Duke, when the Wind was said to be a brave Gale, because it was favourable to us, but would have been called a Storm by Ships to whom that Wind was contrary; Each Wave, as we sailed along, did pass under our Ship, exactly in six Seconds of Time. I did not measure their Bigness; but I am satisfied they look'd and were much bigger, than according to Sir *Isaac Newton's* Theory.

The Swiftnes of the Waves was then such, that the Top of most of them would just begin to offer to break, but could only shew a small Whiteness rowling on the Top of those Waves, keeping an equal pace with the Waves themselves.

In the Theory of Waves, the *Vis Centrifuga* of the Parts of Water must be considered, both when the Curves which they describe turn their Convexity towards Heaven, and also when they turn it towards the Earth. And the various Depth of the Sea must be considered also. It is that *Vis Centrifuga* that does just support those so prodigious Masses of Water above the natural Level of the Sea. And it is the like Force, in a contrary Position and Degree, that does dig continually, between the Waves, those prodigious Furrows, under the same natural Level of the Waters: In which Level the Waves have their *Punctum Flexus contrarii*.

These few Remarks will be sufficient for Others to establish, as I have done elsewhere, the Theory of Waves upon a Mathematical Foundation: To which it will remain to give the last Hand, by a long and accurate Experience.

The Librations of Water in a Tube, as proposed by Sir *Isaac Newton*, have scarce any Reference to the real Theory of Waves: Both because they have no Relation with any *Vis Centrifuga*; And because the Waves working on every Part of the Sea which is in reach of their Agitation, the infinite Perplexity of those imagined Tubes, and their Penetration with one another, would confound all their Operations, and all manner of Mathematical Reasonings about them.

F I N I S.





